Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics FP3 (WFM03)
Paper 01

IAL Mathematics: Further Pure 3 June 2023
Specification: WFM03

Introduction

This paper proved to be a fair test of student knowledge and understanding. There were accessible marks available to all students but there was also plenty of challenge for higher ability students.

Question 1

The opening question on an equation including hyperbolic functions and an exponential term was a good source of marks for almost all students, with a large number of fully correct responses seen. The definitions of sinh and cosh in terms of exponentials were very well-known and most errors were algebraic slips in forming the required the three-term quadratic.

Question 2

This question on finding an inverse matrix and applying it to transform a plane saw quite a mixed response. All elements of the matrix were numerical and so the question required all stages of working to be seen although it was very rare to see responses that just gave an answer. The full method for inverting a 3 by 3 matrix was generally well- recalled although there were of course slips – sometimes with the determinant or the odd error with a minor or cofactor.

Part (b) expected students to use the inverse to transform a point but many did not appreciate the significance of part (a) and proceeded to use the original matrix. This approach led to more algebra and was more prone to error. With either method it was rare to see inappropriate attempts to perform the matrix-vector multiplication. Those who had obtained the correct results in part (b) invariably went on to find the correct equation of the plane in part (c) although sometimes this wasn't given in the required form or with integer coefficients. There were a few long-winded alternative approaches seen here that involved attempts to find the plane equation without using the work already done – these had mixed outcomes. This question part was worth two marks and students would benefit by using the number of marks allocated to give them idea of how much work is likely to be involved if they choose the right route.

Question 3

Question 3 turned out to be quite demanding, particularly in part (a). Almost all attempts obtained the correct derivative and used the correct formula for arc length but dealing with the trigonometric terms eluded many. Those who applied identities first were much more likely to be successful. A few sign errors with identities were seen. Finding the square of the derivative first and then using identities produced a lot of algebra that many students gave up on. There was a requirement to show the result of the integrand in an expanded form – many fell foul of this by rushing to the given answer without dealing with the algebra. A small but significant number of attempts converted to sin and cos at an early stage but were hardly ever able to recover the required terms in tan and cot.

There was more success performing the integration in part (b) although the requirement for identities again was an obstacle to some. Again, some attempts tried to convert to sin and cos but these usually led nowhere. Those who could recognise the integrations needed after applying the identities were often completely successful although a few slips were seen applying the limits.

Question 4

There were only a very small number of completely correct attempts to this vector question although many did make reasonable progress in part (a). This part was a standard task of finding the line of intersection of two planes but it remains the case that this topic area is not well understood by many students. Most were able to obtain plane equations but some efforts stopped abruptly after this. There were two main options – either use algebra on the two equations to form a Cartesian equation which could be converted into the required vector equation, or to find a point on both planes by inspection and then determine the direction, usually by taking the vector product of the normal. The latter was a much more successful option although most attempts used the former and were much more prone to algebraic and methodical slips. The last mark required an equation of the line and it was unfortunate to see otherwise correct responses lose this mark by giving their answer as l = rather than $\mathbf{r}$ =.

It was rare to be awarding many marks in part (b). Because the locations of the vertices of the tetrahedron were not all given, most students drew a blank and were not able to find the vector from C to D by scaling the direction vector so it had a length of 5. Some partial credit was given for those who obtained two vectors for the edges or who gave a correct formula for the volume applied to the given tetrahedron. Of the few who were able to find the right strategy some succumbed to errors with the numbers. A few attempts at long-winded alternatives were seen but they were rarely complete – most of these attempted the distance to one of the planes from one of the vertices but were usually abandoned shortly afterwards.

Question 5

Although there was some good scoring on this question on eigenvalues it was quite unusual to see responses that were fully correct. The method for obtaining a relevant equation in lambda was well known although there were many algebraic slips seen. Having obtained a cubic equation, some students were not sure where to go next, but many were able to see that reducing it to a quadratic was the way forward. Most responses picked up on the “repeated” eigenvalue mentioned in the question and usually applied the discriminant to find a value for k. Other variations were seen that used the sum and product of the roots of the quadratic. Very few were able to identify that the other alternative was if the constant in the quadratic was zero. The very few who did apply both of these approaches sometimes failed to give the corresponding eigenvalues at the end or made sign errors with them.

Question 6

Good scoring was seen on this ellipse question but the requirement for a locus in part (c) was fairly discriminating.

Part (a) was a standard "bookwork" question and most were prepared for it. Differentiation was usually done parametrically rather than implicitly and explicit attempts were happily very rare. Many achieved a correct line equation but students ought to be aware that questions like this often require an intermediate step before the final answer is given.

Part (b) required a normal and most knew to apply the perpendicular gradient rule. As is often the case with line formation questions, a $y = mx + c$ approach is not usually the best choice and requires extra work that sometimes lets in slips.

Progress was more mixed in the final part. Because the question involved a tangent, a normal and a midpoint, some responses became confused putting the information together coherently by confusing their lines or forgetting to find a midpoint altogether. Those who had navigated the first part of the question well usually realised the need to apply the Pythagorean trig identity to obtain the required equation. Slips were quite common though, with many unable to give the exact form that was specified in the question.

Question 7

This reduction formula question had a fairly standard part (a) which had the usual mixed response and a slightly unusual part (b) which only the best students could come through with all the marks.

Many were well used to the method with this type of integral and a lot of fully correct responses were seen with the more confident students. Way 1 was by far the sensible choice – a very small number of attempts via Way 2 were seen and these were rarely able to split the resulting integrand to get it into an appropriate form suitable for integration. A significant number of attempts failed to apply the chain rule when differentiating $\cosh^{n-1}2x$. Although invisible brackets were condoned if they were recovered before the given answer and missing "dx"s were allowed throughout, there was no tolerance of incorrect arguments here or e.g., cosh lazily written as cos.

In part (b) most were able to perform the required expansion although there were a surprising number of errors with this fairly straight-forward task. Most went on to apply the reduction formula although there were some unfortunate slips with its application seen on a regular basis. A recurring error was to think that I_0 was 0 rather than x. Those who were able to get the correct form for the required integrations were usually able to put them together to form an appropriate expression. However, some did not collect like terms. Some attempts unnecessarily applied further identities which left them vulnerable to losing the last accuracy mark. It was pleasing to see that attempts via direct integrations rather than via the formula were extremely rare.

Question 8

The most confident students made short work of this question involving the differentiation and integration of an arcosh function but scoring on the whole was not widespread. Most did use the product rule in part (a) but despite the derivative of arcosh being in the formula book there were some disappointing slips here. Particularly common was to lose the “5” from the numerator of the fraction but there were some follow-through accuracy marks allowed for this and a few other minor errors. A few attempts converted to the logarithmic form before differentiation and this is rarely a sensible idea.

Part (b) required rearrangement of the result in part (a) but it was more common to see parts applied to 1 x arcosh 5x. The resulting integral was not recognised widely but many making it this far confidently produced a fully correct expression. Use of substitutions was unusual and often unsuccessful. Limit application was usually appropriate and the logarithmic form of arcosh was generally applied correctly with both values. The answer was not a fully given one but students who were able to deduce it following incorrect work could not access the last two accuracy marks.